On Uncertainty and Robustness in Evolutionary Optimization-Based Multi-Criterion Decision-Making

Doctoral Dissertation

By

Daniel E. Salazar Aponte

Under the Supervision of

Blas Galván & Claudio Rocco
Acknowledgement

The activities carried out during this research were granted by

- Ceani, Siani, ULPGC
- Consejería de Educación
- Fundación Universitaria Las Palmas
What is this thesis about?

Multi-Criterion
Decision-Making

Evolutionary
Algorithms

Uncertainty

Dependability
Decision-Making

Alternatives

Decision Makers

\[ X = \text{arg opt} [F(x)] \]
\[ \text{s.t. } G(x) \leq 0 \]
Evolutionary Algorithms

Sample → Population → $F_0(F'(x))$

Recombination

$y_2$

$y_1$

$y_3$

$x_1$

$x_2$

$x_3$

$x_7$

$x$

$y_0$

$x_1$

$x_2$
Effect of Uncertainty

Epistemic Case

Aleatory Case
What people from EMO have done?

- Teich, Hughes
- Sørensen, Jin et al.
- Limbour, Fieldsen et al., Deb et al.
- Barrico et al., Basseur et al.
- Deb et al.

1. CEC 2005
2. MOPGP06
3. IJNKM, Ress, Rams, ESREL, 2007
4. Vulnerability applications
AUREO: Analysis of Uncertainty and Robustness in Evolutionary Optimization

1st Stage: Find a suitable mathematical program
(I have identified 3 classes of problems
(according to the available information)

2nd Stage: Define the algorithmic structure
(I have made proposals for classes 1 and 2 and
I have solved class 2, 3 dependability problems
using NSGA-II and MOSA)
Classes of Decision-Making Problems

Under Uncertainty

\[
\begin{align*}
F'(x, p) &\rightarrow Y \\
\text{OPT} \left[ R(F'(x, p), x, p, \delta_x, \delta_p) \right] \\
5.7. \ G(x) &\leq 0 \\
\delta_x &\leq \delta_x' \leq \delta_x ; \delta_p &\leq \delta_p' \leq \delta_p \\
I(F'(x, p), x, p, \delta_x, \delta_p, r)
\end{align*}
\]
Theories About Uncertainty (1/3):

- **Element**
- **Bound**

**Set:** Collection of elements

**Interval:** Continuous set

Fuzzy Logic: The Bound is Blurred

- Some elements belong, others do not
- Others “more or less”

Possibility Theory:

- Bivalent logic
- Graded logic
Theories about Uncertainty (2/3):

Probability Theory: we make statements about likelihoods

- Bound well defined, all elements can occur
  \[ 0 \leq p \leq 1; \sum_i p_i = 1; P(S) = 1 \]

Imprecise Probabilities: different approaches like Walley's, DS's or P-boxes

- \[ 0 \leq p \leq 1; \sum_i p_i \geq 1 \]
Theories about Uncertainty (3/3):

- All of them allow us to define representative quantities.

- All of them prescribe propagation techniques.

\[ X \rightarrow [F(x)] \rightarrow Y \]

- Extension principle
- Interval arithmetic
- Convolution
- Monte Carlo
**Class 1:**

*Input:* $S_x, S_p$

**Information Constraint $I(\cdot)$:** Possibly

<table>
<thead>
<tr>
<th>Check for Optimality:</th>
<th>Control Density:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Means</td>
<td>Use Indicators</td>
</tr>
<tr>
<td>Variances</td>
<td>Contrast Means</td>
</tr>
<tr>
<td>Percentiles</td>
<td>Prob Density</td>
</tr>
<tr>
<td>Extreme Values</td>
<td></td>
</tr>
</tbody>
</table>

**Non-Prob Heuristics**

**IP-MOEAs**
Class 2:

Input: $\delta_x, \delta_y$?

Information constraint $I(\cdot)$: Mandatory

Transform the problem:

Maximize the "box" around $x$

Given that $I(\cdot)$ is met by $y$
Class 3:

Input: $\delta_x, \delta_y$?

Constraint: $I(\cdot)$?

→ Elicitate $\delta_x, \delta_p$ and get a Class 1

or

→ Suppose $\delta_x, \delta_p$ just to explore $Y$ to help the DM to define $I(\cdot)$
Dependability Application Examples

NPP Robust Design

Scheduling of a Waste Treatment System

Vulnerability Analysis
NPP Robust Design

Robust Design:

Maximise the box for $R_i$ so that $0.9 \leq R_5 \leq 1$
Scheduling Problem

1) This is a class 3 problem
2) We assumed $\delta_x, \delta_p$
3) We explored $Y$
4) We set $J(\cdot)$
5) We solved
Vulnerability Analysis

-What if someone attack?

Analyzing one point

What point should we protect:
\[\Rightarrow \text{max-min impact}\]

Robust protection
WRAPPING UP

In this thesis we have:

- Developed a diagnosis tool based on classes of MCDM problems: AUREO
- Proposed new algorithmic designs or improvements
- Solved dependability problems using AUREO
On the future work:

- **Implement and test the algorithms proposed for Class 1**
- **Continue working on vulnerability analysis**
- **Apply AUReo on new problems**
Thank you!

It's time for questions...